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Inherent regenerative losses of a ferroelectric Ericsson refrigeration cycle

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Abstract

The performance of a ferroelectric Ericsson refrigeration cycle is investigated on the basis of the statistic relation between the electrical polarization and the electric field strength of the ferroelectric materials. The inherent regenerative losses in the cycle are calculated. The coefficients of performance of the cycle are derived. Moreover, the performance of the Ericsson refrigeration cycle using other dielectric materials as the working substance is discussed. The results obtained here may reveal the general characteristics of the electrocaloric Ericsson refrigeration cycle.

Keywords: Ericsson refrigeration cycle; Ferroelectric material; Regenerative loss; Coefficient of performance

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1. Introduction

Like magnetic refrigeration [1–5], electrocaloric refrigeration is another new refrigeration technique [6]. The difference is that in electrocaloric refrigeration, the entropy of the working substance is changed by an electric field rather than by a magnetic field. Although the first experiments with the electrocaloric effect were on Rochelle salt by Kobeko and Kurtschatov in 1930 [7], the concepts of the Carnot refrigeration cycle and other cryogenic refrigeration cycle based on the electrocaloric effect were put forward until 1970s [7,8]. Since then, an increasing interest has been paid to low temperature devices based on the electrocaloric effect [6,9,10].

It is well known that the practical use of electrocaloric refrigeration is not only dependent on discovery of suitable ferroelectric materials, but also associated with the method of a refrigeration cycle. The Ericsson refrigeration cycle is a type of thermodynamic cycle. When a ferroelectric material is used as the refrigerant, the Ericsson refrigeration cycle consists of two isothermal and two constant electric field processes. It is controlled more easily than other ferroelectric refrigeration cycles such as the Stirling refrigeration cycle composed of two isothermal and two constant polarization processes. Thus, it is of significance to research the performance of the ferroelectric Ericsson refrigeration cycle.

In the present paper, the main purpose is to analyze the influence of the inherent regenerative losses on the performance of the ferroelectric Ericsson refrigeration cycle and reveal the general characteristics of the cycle.

2. Thermodynamic properties of ferroelectric materials

To begin with, let us consider ferroelectric materials in a unit volume and assume that their volume change is negligible. Then, the fundamental equation of thermodynamics of ferroelectric materials may be expressed as [11–14]

$$\mathrm{d}u = T\,\mathrm{d}s + (E + \beta P)\,\mathrm{d}P \tag{1}$$

where u, s, and P are, respectively, the internal energy, entropy, and electrical polarization of ferroelectric materials per unit volume, E is the electric field strength, T is the absolute temperature, and β is a parameter which is independent of temperature.

Using Eq. (1), one can obtain

$$\left(\frac{\partial s}{\partial P}\right)_T = -\left(\frac{\partial E}{\partial T}\right)_P \tag{2}$$

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Nomenclature

C	Curie constant	
C_E	specific heat at constant electric field strength	$\mathrm{J{\cdot}K^{-1}}$
C_P	specific heat at constant electrical polarization	$\mathrm{J{\cdot}K^{-1}}$
E	electric field strength	$\mathrm{V{\cdot}m^{-1}}$
E_1	high electric field strength	$\mathrm{V{\cdot}m^{-1}}$
E_2	low electric field strength	$\mathrm{V{\cdot}m^{-1}}$
k	Boltzmann's constant	$\mathrm{J{\cdot}K^{-1}}$
N	number of electric dipole per unit volume	
P	electrical polarization	$\mathrm{C{\cdot}m^{-2}}$
Q_{bc}	heat transferred into the regenerator at high constant electric field process	J
Q_{da}	heat transferred out the regenerator at low constant electric field process	J
Q_c	heat absorbed from the hot reservoir at high isothermal process	J
Q_h	heat released to the cold reservoir at low isothermal process	J
ΔQ	net heat in the regenerator	J
s	entropy of ferroelectric materials per unit volume	$\mathrm{J{\cdot}K^{-1}}$
s_0	entropy of ferroelectric materials per unit volume when $P = 0$	$\mathrm{J{\cdot}K^{-1}}$
T	absolute temperature	K
T_0	Curie temperature	K
T_c	temperature of the cold reservoir	K
T_h	temperature of the hot reservoir	K
T_m	temperature in which the entropy difference of the two constant electric field curves is maximum	K
u	internal energy of ferroelectric materials per unit volume	J
W	input work	J

Greek symbols

β	a parameter which is independent of temperature	$\mathrm{V{\cdot}m{\cdot}C^{-1}}$
ε	coefficient of performance	
ε_c	coefficient of performance of a Carnot refrigerator	
μ	moment of electric dipole	$\mathrm{J{\cdot}m{\cdot}V^{-1}}$

and

$$\begin{aligned} C_E &= T\left(\frac{\partial s}{\partial T}\right)_E = T\left(\frac{\partial s}{\partial T}\right)_P + T\left(\frac{\partial s}{\partial P}\right)_T\left(\frac{\partial P}{\partial T}\right)_E \\ &= C_P - T\left(\frac{\partial E}{\partial T}\right)_P\left(\frac{\partial P}{\partial T}\right)_E \end{aligned} \tag{3}$$

where $C_E = T(\frac{\partial s}{\partial T})_E$ and $C_P = T(\frac{\partial s}{\partial T})_P$ are the specific heat at constant electric field strength and the specific heat at constant electrical polarization, respectively. Starting from Eqs. (2) and (3), one can derive the concrete expressions of the thermodynamic quantities of various ferroelectric materials as long as the equations of state of ferroelectric materials are known.

Based on Boltzmann's statistics theory, the polarization may be given as [15]

$$P = N\mu \tanh\frac{\mu(E+\beta P)}{kT} \tag{4}$$

where N is the number of electric dipole per unit volume, μ is the moment of electric dipole, and k is the Boltzmann's constant.

For the sake of convenience, Eq. (4) may be expressed as

$$E = -\beta P + \frac{kT}{\mu}\tanh^{-1}\left(\frac{P}{N\mu}\right) \tag{5}$$

Substituting Eqs. (4) and (5) into Eqs. (2) and (3), we obtain

$$s = s_0(T) - \frac{P(E+\beta P)}{T} - \frac{1}{2}Nk\ln\left[1 - \left(\frac{P}{N\mu}\right)^2\right] \tag{6}$$

and

$$\begin{aligned} C_E &= C_P - (E+\beta P)\left(\frac{\partial P}{\partial T}\right)_E \\ &= C_P - \left\{\left(N\mu^2\right)/\left(kT^2\right)(E+\beta P)^2 \right. \\ &\qquad \left. \times \sec h^2\left[\mu(E+\beta P)/(kT)\right]\right\} \\ &\qquad \times \left\{\frac{(N\mu^2\beta)}{(kT)\sec h^2\left[\frac{\mu(E+\beta P)}{(kT)}\right]} - 1\right\}^{-1} \end{aligned} \tag{7}$$

where $s_0(T)$ is the entropy of the ferroelectric system when $P = 0$ and is only a function of temperature. Its value depends on the properties of the concrete ferroelectric material.

Using Eqs. (1) and (5), we can prove

$$\left(\frac{\partial u}{\partial P}\right)_T = E + \beta P - T\left(\frac{\partial E}{\partial T}\right)_P = 0 \tag{8}$$

Integrating Eq. (8) gives

$$u = u(T) \tag{9}$$

which is only a function of temperature. It is clearly seen from Eq. (9) that the specific heat C_P at constant electrical polarization is only a function of temperature. However, the specific heat C_E at constant electric field strength not only is a function of temperature but also depends closely on the electric field strength.

3. A ferroelectric Ericsson refrigeration cycle

Fig. 1 shows a schematic diagram of the Ericsson refrigeration cycle. In order to improve the performance of the cycle, a regenerator is often applied in two constant electric field processes. In Fig. 1, Q_h and Q_c are the amounts of heat exchange between the working substance and the heat reservoirs at temperatures T_h and T_c during the two isothermal processes, E_1 and E_2 are the high and low electric fields, and Q_{bc} and Q_{da} are the amounts of heat exchange between the working substance and the regenerator during the constant electric field processes. All the heats Q_h, Q_c, Q_{bc}, and Q_{da} are positive.

Using Eq. (6), one can obtain the amounts of heat exchange in two isothermal processes

$$\begin{aligned} Q_h &= T_h(s_a - s_b) \\ &= P_b(E_1 + \beta P_b) - P_a(E_2 + \beta P_a) + \frac{1}{2} N k T_h Y_1 \end{aligned} \tag{10}$$

and

$$\begin{aligned} Q_c &= T_c(s_d - s_c) \\ &= P_c(E_1 + \beta P_c) - P_d(E_2 + \beta P_d) + \frac{1}{2} N k T_c Y_2 \end{aligned} \tag{11}$$

respectively, where s_a, s_b, s_c, and s_d are the entropies of the system in the states a, b, c, and d shown in Fig. 1,

$$Y_1 = \ln\left[\frac{1 - (P_b/N\mu)^2}{1 - (P_a/N\mu)^2}\right]$$

and

$$Y_2 = \ln\left[\frac{1 - (P_c/N\mu)^2}{1 - (P_d/N\mu)^2}\right]$$

Using Eq. (7), one can derive the amount of heat exchange in two constant electric field processes

$$\begin{aligned} Q_{bc} &= \int_c^b C_E \, \mathrm{d}T \\ &= \int_{T_c}^{T_h} C_P(T) \, \mathrm{d}T + E_1(P_c - P_b) + \frac{\beta}{2}\left(P_c^2 - P_b^2\right) \end{aligned} \tag{12}$$

and

$$\begin{aligned} Q_{da} &= \int_d^a C_E \, \mathrm{d}T \\ &= \int_{T_c}^{T_h} C_P(T) \, \mathrm{d}T + E_2(P_d - P_a) + \frac{\beta}{2}\left(P_d^2 - P_a^2\right) \end{aligned} \tag{13}$$

respectively, where $P_a = P(E_2, T_h)$, $P_b = P(E_1, T_h)$, $P_c = P(E_1, T_c)$, and $P_d = P(E_2, T_c)$.

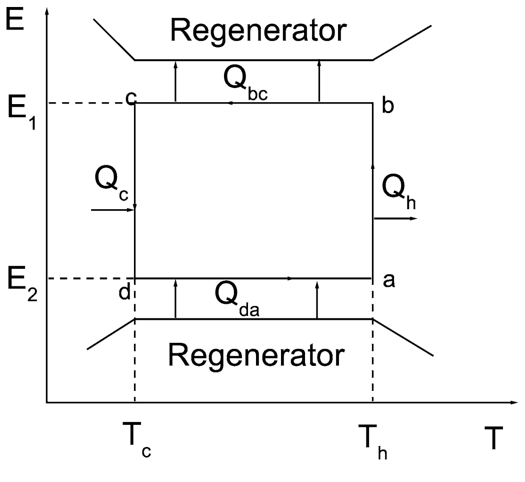


Fig. 1. The field-temperature schematic diagram of a ferroelectric Ericsson refrigeration cycle. The units of electric field strength and temperature are $\mathrm{V \cdot m^{-1}}$ and J, respectively.

From Eqs. (10)–(13), one obtain the input work per cycle as

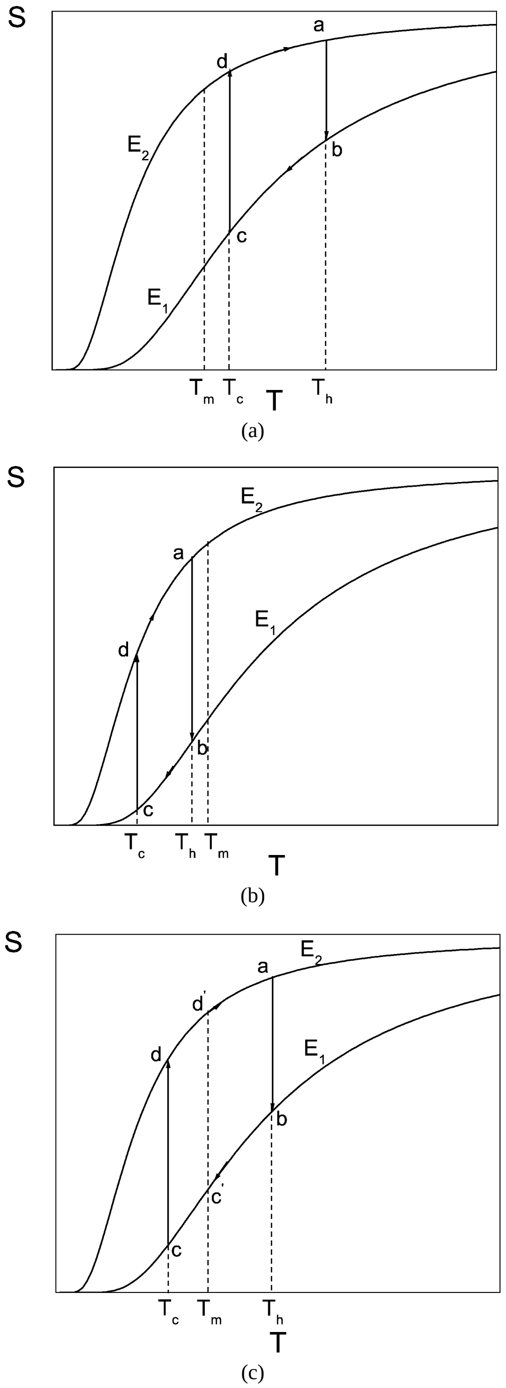
$$\begin{aligned} W &= Q_h - Q_c + Q_{bc} - Q_{da} \\ &= \frac{1}{2}\beta\left(P_d^2 - P_c^2 + P_b^2 - P_a^2\right) + \frac{1}{2} N k (T_h Y_1 - T_c Y_2) \end{aligned} \tag{14}$$

Using the cycle model mentioned above and the fundamental equations, we can discuss the regenerative characteristics of the ferroelectric Ericsson refrigeration cycle and other performances.

4. The regenerative characteristics

For a ferroelectirc Ericsson refrigeration cycle operating only between two reservoirs at temperatures T_h and T_c, the temperature of the working substance in the constant electric field processes is always lower than that of the hot reservoir and higher than that of the cold reservoir. When the heat transferred into the regenerator is larger than that transferred out of the regenerator in two constant electric field processes, the redundant heat in the regenerator can only released to the cold reservoir in a timely manner. If not, the temperature of the regenerator would be changed such that the regenerator would not operate normally. Similarly, when the heat transferred into the regenerator is smaller than that transferred out of the regenerator in the two constant electric field processes, the inadequate heat in the regenerator can only be compensated from the hot reservoir in a timely manner.

According to the entropy-temperature diagram [7] of a ferroelectric material, there may exist three different cases for the ferroelectirc Ericsson refrigeration cycle, as shown in Fig. 2(a), (b), and (c), where T_m is the temperature in which the entropy difference of the two constant electric field curves is the maximum. It is worthwhile to note that the temperature T_m may or may not equal to the Curie temperature. The value of T_m depends on the properties of



(a)

(b)

(c)

Fig. 2. The entropy-temperature diagrams of a ferroelectric Ericsson refrigeration cycle. (a) $T_h > T_c \geqslant T_m$, (b) $T_m \geqslant T_h > T_c$, and (c) $T_h > T_m > T_c$. The units of entropy and temperature are $\mathrm{J{\cdot}K^{-1}}$ and K, respectively.

the concrete ferroelectric material as well as the magnitudes of E_1 and E_2.

In Fig. 2(a), $T_h > T_c > T_m$ and the Ericsson refrigeration cycle is in the convergent region of the s–T diagram. It may be proven the heat transferred into the regenerator is larger than that transferred out of the regenerator in two constant electric field processes, i.e.,

$$\Delta Q = Q_{bc} - Q_{da} > 0 \tag{15}$$

The redundant heat in the regenerator can only be released to the cold reservoir. This results in the reduction of the refrigeration heat from Q_c to Q_c'. From Eqs. (11)–(13), one can calculate the refrigeration heat

$$\begin{aligned} Q_c' &= Q_c - \Delta Q \\ &= \frac{1}{2} N k T_c Y_2 - \Big[E_2 P_a - E_1 P_b \\ &\qquad + \frac{\beta}{2}\big(P_d^2 - P_c^2 + P_a^2 - P_b^2\big)\Big] \end{aligned} \tag{16}$$

From Eqs. (14) and (16), one can conclude when the ferroelectric Ericsson refrigeration cycle is operated in the region of $T_h > T_c > T_m$, the coefficient of performance is given by

$$\begin{aligned} \varepsilon &= \frac{Q_c'}{W} \\ &= \Big\{ \frac{1}{2} N k T_c Y_2 - \Big[E_2 P_a - E_1 P_b \\ &\qquad + \frac{\beta}{2}\big(P_d^2 - P_c^2 + P_a^2 - P_b^2\big)\Big]\Big\} \\ &\quad \times \Big[\frac{1}{2}\beta\big(P_d^2 - P_c^2 + P_b^2 - P_a^2\big) \\ &\qquad + \frac{1}{2} N k (T_h Y_1 - T_c Y_2)\Big]^{-1} < \varepsilon_c \end{aligned} \tag{17}$$

where $\varepsilon_c = T_c/(T_h - T_c)$ is the coefficient of performance of a reversible Carnot refrigerator.

In Fig. 2(b), $T_m > T_h > T_c$ and the Ericsson refrigeration cycle is in the divergent region of the s–T diagram. In such a case, the heat transferred into the regenerator is smaller than that transferred out of the regenerator in the two constant electric field processes, and the inadequate heat in the regenerator,

$$\Delta Q = Q_{bc} - Q_{da} < 0 \tag{18}$$

can be compensated from the hot reservoir, while Q_c is unvarying. From Eqs. (11) and (14), one may find that the coefficient of the performance of a ferroelectric Ericsson refrigeration cycle operated in the region of $T_m > T_h > T_c$ is given by

$$\begin{aligned} \varepsilon &= \frac{Q_c}{W} \\ &= \frac{P_c(E_1 + \beta P_c) - P_d(E_2 + \beta P_d) + \frac{1}{2} N k T_c Y_2}{\frac{1}{2}\beta(P_d^2 - P_c^2 + P_b^2 - P_a^2) + \frac{1}{2} N k (T_h Y_1 - T_c Y_2)} < \varepsilon_c \end{aligned} \tag{19}$$

When $T_h > T_m > T_c$, there are three possible cases: (a) $\Delta Q > 0$, (b) $\Delta Q < 0$, and (c) $\Delta Q = 0$, as shown in Fig. 2(c). In any case, when the temperature of the working substance in the regeneration process is lower than T_m, the heat transferred into the regenerator is smaller than that transferred out of the regenerator. The inadequate heat in the regenerator can only be compensated from the hot reservoir in a timely manner. While the temperature of the working substance in the regeneration process is higher than T_m, the heat transferred into the regenerator is larger than that transferred out of the regenerator. The redundant heat in the regenerator can only be released to the cold reservoir in the timely manner. This results in the reduction of the regeneration heat from Q_c to Q_c''. Using Eqs. (4) and (7), we can obtain that when $T_h > T_m > T_c$, the heat $\Delta Q'$ released to the cold reservoir in the regeneration process per cycle is

$$\begin{aligned}\Delta Q' &= Q_{bc'} - Q_{d'a}\\ &= E_1(P_{m1} - P_b) - E_2(P_{m2} - P_a)\\ &\quad + \frac{\beta}{2}\left(P_{m1}^2 - P_b^2 + P_a^2 - P_{m2}^2\right) \qquad (20)\end{aligned}$$

where $P_{m1} = P(E_1, T_m)$ and $P_{m2} = P(E_2, T_m)$. Thus, the refrigeration heat Q_c'' may be expressed as

$$\begin{aligned}Q_c'' &= Q_c - \Delta Q'\\ &= \frac{1}{2}NkT_cY_2 + E_1(P_c + P_b - P_{m1})\\ &\quad - E_2(P_d + P_a - P_{m2})\\ &\quad + \frac{\beta}{2}\left(2P_c^2 + P_b^2 - 2P_d^2 - P_a^2 - P_{m1}^2 + P_{m2}^2\right) \qquad (21)\end{aligned}$$

It is worthwhile to note that case (c) may not be perfect regeneration although $\Delta Q = 0$, because the redundant and inadequate heats in the regeneration process are respectively released to the cold reservoir and compensated from the hot reservoir in the different times but do not cancel each other out.

From Eqs. (14) and (21), one may conclude that the coefficient of performance of the ferroelectric Ericsson refrigeration cycle operated in the region of $T_h > T_m > T_c$ is given by

$$\begin{aligned}\varepsilon &= \frac{Q_c''}{W}\\ &= \big[(1/2)NkT_cY_2 + E_1(P_c + P_b - P_{m1})\\ &\quad - E_2(P_d + P_a - P_{m2})\\ &\quad + (\beta/2)\left(2P_c^2 + P_b^2 - 2P_d^2 - P_a^2 - P_{m1}^2 + P_{m2}^2\right)\big]\\ &\quad \times \big[(1/2)Nk(T_hY_1 - T_cY_2)\\ &\qquad + (\beta/2)\left(P_d^2 - P_c^2 + P_b^2 - P_a^2\right)\big]^{-1} < \varepsilon_c \qquad (22)\end{aligned}$$

Equation (22) shows that when $T_h > T_m > T_c$, the non-perfect regeneration also results in the reduction of the coefficient of performance of the ferroelectric Ericsson refrigerator.

Finally, it may be concluded that for the Ericsson refrigeration cycle using ferroelectric materials as the working substance, due to the non-perfect regeneration, its coefficient of performance is always less than that of a reversible Carnot refrigeration cycle.

5. Two special cases

(1) $\beta = 0$. When the additional electric field produced by the polarization is negligible, $\beta = 0$ and Eq. (4) may be simplified as

$$P = N\mu \tanh\frac{\mu E}{kT} \qquad (23)$$

In this case, the coefficients of performance for the ferroelectric Ericsson refrigeration cycle,

$$\begin{aligned}\varepsilon &= \frac{Q_c'}{W} = \frac{\frac{1}{2}NkT_cY_2 - (E_2P_a - E_1P_b)}{\frac{1}{2}Nk(T_hY_1 - T_cY_2)} < \varepsilon_c\\ &\quad (T_h > T_c > T_m) \qquad (24)\\ \varepsilon &= \frac{Q_c}{W} = \frac{P_cE_1 - P_dE_2 + \frac{1}{2}NkT_cY_2}{\frac{1}{2}Nk(T_hY_1 - T_cY_2)} < \varepsilon_c\\ &\quad (T_m > T_h > T_c) \qquad (25)\end{aligned}$$

and

$$\begin{aligned}\varepsilon = \frac{Q_c''}{W} &= \big[(1/2)NkT_cY_2 + E_1(P_c + P_b - P_{m1})\\ &\quad - E_2(P_d + P_a - P_{m2})\big]\\ &\quad \times \big[(1/2)Nk(T_hY_1 - T_cY_2)\big]^{-1} < \varepsilon_c\\ &\quad (T_h > T_m > T_c) \qquad (26)\end{aligned}$$

can be deduced directly from Eqs. (17), (19), and (22), respectively.

(2) *At high temperatures.* At high temperatures,

$$\frac{\mu(E + \beta P)}{kT} \ll 1 \quad \text{and} \quad \frac{\mu E}{kT} \ll 1$$

Eqs. (4) and (23) returns to the Curie–Weiss law [16–19]

$$P = \frac{CE}{T - T_0} \qquad (27)$$

and the Curie law [14,16,17,19]

$$P = CE/T \qquad (28)$$

respectively, where $T_0 = N\mu^2\beta/k$ is the Curie temperature and $C = N\mu^2/k$ is a constant characteristic of the ferroelectric materials called the Curie constant. It has been proven experimentally that the properties of a number of dielectric materials obey the Curie–Weiss law or the Curie law. This implies the fact that when these materials are used as the working substance, the coefficient of performance of a ferroelectric Ericsson refrigeration cycle

$$\varepsilon = \frac{Q_c'}{W} = \frac{T_c}{T_h - T_c} - \frac{T_h}{T_h - T_0} < \varepsilon_c \qquad (29)$$

and

$$\varepsilon = \frac{T_c}{T_h - T_c} - 1 < \varepsilon_c \qquad (30)$$

can be directly derived from Eqs. (17) and (24), respectively. It is of interest to note the fact that Eqs. (29) and (30) are identical with the expressions of the coefficients of performance [1,2] for the Ericsson refrigeration cycle of paramagnetic salt which obeys the Curie–Weiss law and the Curie law.

6. Discussion

In order to make the results more general, we will further analyze the performance of the Ericsson refrigeration cycle using other dielectric materials as the refrigerant.

(1) For the antiferroelectric crystals, the polarization may be expressed as [19]

$$P = CE/(T + T_0) \tag{31}$$

which cannot derived directly from Eq. (4). Comparing Eq. (31) with Eq. (27), we can find without difficulty that when the working substance obeys Eq. (31), the coefficient of performance of the Ericsson refrigeration cycle is

$$\varepsilon = \frac{T_c}{T_h - T_c} - \frac{T_h}{T_h + T_0} < \varepsilon_c \tag{32}$$

(2) For a wide class of dielectric materials with ionic and polar covalent bonds, the relation between P and E may be expressed as [19,20]

$$P = \frac{CE}{T_0 - T} \tag{33}$$

Although this relation has not been described in the physics of dielectrics, it is a very interesting relation which has been experimentally verified [19]. Using the Similar method mentioned above, one can derive the coefficient of performance of the Ericsson refrigeration cycle (a detailed derivation is given in the appendix) as

$$\varepsilon = \frac{Q_c}{W} = \frac{T_c}{T_h - T_c}\frac{T_0 - T_h}{T_0 - T_c} \tag{34}$$

7. Conclusions

It can be clearly seen from the results obtained above that the ferroelectric Ericsson refrigeration cycle is analogous to a ferromagnetic Ericsson refrigeration cycle. In general, it does not possess the condition of perfect regeneration. Its coefficient of performance is always less than that of a Carnot refrigeration cycle for the same temperature range. However, the ferromagnetic Ericsson refrigeration cycle may have more actual application than the Carnot refrigeration cycle. Moreover, it has been proven that for the Ericsson refrigeration cycle using other dielectric materials mentioned above as the working substance, there also exist some similar characteristics.

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Appendix A

Starting from Eq. (33) and using the similar method mentioned in this paper, we can derive

$$Q_h = \frac{C(E_1^2 - E_2^2)}{2}\frac{T_h}{(T_0 - T_h)^2}$$

$$Q_c = \frac{C(E_1^2 - E_2^2)}{2}\frac{T_c}{(T_0 - T_c)^2}$$

$$\begin{aligned}
Q_{bc} &= \int_c^b C_E\,\mathrm{d}T = \int_{T_c}^{T_h} C_P(T)\,\mathrm{d}T + \int_{T_c}^{T_h} \frac{CTE_2^2}{(T_0 - T)^3}\,\mathrm{d}T \\
&= \int_{T_c}^{T_h} C_P(T)\,\mathrm{d}T \\
&\quad - CE_2^2\left[\frac{1}{T_0 - T_h} - \frac{1}{T_0 - T_c} - \frac{T_0}{2(T_0 - T_h)^2} + \frac{T_0}{2(T_0 - T_c)^2}\right]
\end{aligned}$$

and

$$\begin{aligned}
Q_{da} &= \int_d^a C_E\,\mathrm{d}T = \int_{T_c}^{T_h} C_P(T)\,\mathrm{d}T + \int_{T_c}^{T_h} \frac{CTE_1^2}{(T_0 - T)^3}\,\mathrm{d}T \\
&= \int_{T_c}^{T_h} C_P(T)\,\mathrm{d}T \\
&\quad - CE_1^2\left[\frac{1}{T_0 - T_h} - \frac{1}{T_0 - T_c} - \frac{T_0}{2(T_0 - T_h)^2} + \frac{T_0}{2(T_0 - T_c)^2}\right]
\end{aligned}$$

Because $T_0 > T_h > T_c$, we can obtain

$$\begin{aligned}
\Delta Q &= Q_{bc} - Q_{da} \\
&= \frac{C(E_1^2 - E_2^2)}{2}\frac{[2T_hT_c - T_0(T_h + T_c)](T_h - T_c)}{(T_0 - T_h)^2(T_0 - T_c)^2} < 0
\end{aligned}$$

and

$$\begin{aligned}
W &= Q_h - Q_c + \Delta Q \\
&= \frac{C(E_1^2 - E_2^2)}{2}\frac{T_h - T_c}{(T_0 - T_h)(T_0 - T_c)}
\end{aligned}$$

According to the definition of the coefficient of performance, we can derive Eq. (34) from the above equations.

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